A SEQUENCING PROBLEM FOR MIXED-MODEL ASSEMBLY LINE WITH THE HELP OF RELIEF-MAN

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ABSTRACT
There usually exist large variations of the assembly times on mixed-assemble lines depending on the difference of product-models. To increase efficiency of line handling under such circumstance, this paper concerns with a sequencing problem for mixed-model assembly line where each product is assembled within the same cycle time. Then, we formulate a new type of the sequencing problem minimizing the weighted sum of the line stoppage times and the idle times and propose a new sequencing method with the help of Relief Man (RM). Since the resulting problem refers to a combinational optimization problem, we develop a hierarchical method that applies meta-heuristics like SA (Simulated Annealing) together with NNM (Nearest Neighbor Method). Finally, we examine effectiveness of the proposed method through computer simulations and emphasize the flexibility of using RM against various changes of production environment.

Keywords: Mixed-model, Assembly line, Sequencing, Relief-man, Meta-heuristics.

INTRODUCTION
Sequencing is recognized as an important aspect for raising the efficiency of line handling of the assembly line where mixed-models are assembled every constant cycle time. In such mixed-model sequencing problem, one of the two major goals is to level the workload at each workstation on the assembly line against product-models having different assembly time (Miltonburg, 1989). Another one is to keep the constant usage rate of every part at the assembly line (Duplag and Bragg, 1998). Concerns about these two goals have been widely discussed in the literatures. For example, the workload-leveling problem was addressed by Okamura and Yamashina (1978), Yano and Rachamadugu (1991) concerned with the problem minimizing the risk of assembly line stop. Sumichrast and Russell (1990) discussed the parts-usage smoothing problem. Moreover, the problem to attain these two goals simultaneously was discussed by Korkmazel and Meral (2001). However, all these studies are not applicable in the case where large variations exist in the assembly times among different product-models. Tamura (1999) dealt with such situation by means of installing a bypass line. By that, he tried to improve the inefficiency such as line stoppage or idle time of workers due to the difference in the assembly times. However, this approach causes such a disadvantage that we need to remove the installation after it becomes unnecessary.

In this paper, we concern with a sequencing problem in the mixed-model assembly line that achieves the above two goals while overcoming such disadvantage. That is, instead of installation of the bypass line as a hard facility, we will introduce RM (Relief Man) as a soft facility. After formulating the problem, we will propose its practical solution method that can solve the combinational optimization regarding injection sequence and travelling routes of RM. Then, we develop a hybrid method that employs meta-heuristics like SA(Simulated Annealing) and NNM(Nearest Neighbor Method) in a hierarchical manner. Effectiveness of the proposed method is verified through some numerical experiments.

FORMULATION BY AN OPTIMIZATION MODEL
Figure 1 shows an example of the mixed-model assembly line where K workstations are linked by a conveyor moving at constant speed. Each product is fed to the assembly line from the first workstation every interval of cycle time (CT).

We further assume the following conditions.

1. Total number of order is I.
2. The maximum number of parts used on the workstation is M.
3. There are two kinds of worker, i.e., general worker (called just worker hereafter) and RM.
4. Workers are confined to their workstation k(= 1, ..., K) during assembly, and their working time do not exceed CT.
5. When Worker do the work exceeding CT, RM supports them.
6. RM (t = 1, 2, ..., B) can cover all workstations, but cannot operate more than two workstations simultaneously.

7. Line stoppage occurs due to the part shortage supplied from sub-lines and work delays of RM as well. However, these malfunctions do not happen simultaneously on the same workstation.

8. Idle time occurs when the assembly time of RM or worker is less than CT.

After all, our sequencing problem can be formulated as follows:

\[
\begin{align*}
\min_{\pi \in \Pi} \quad & z = \rho \times \sum_{t=1}^{T} \left( \max_{t \in \cal{K}} \left( P_k^t, \sum_{b=1}^{N^t} X_{b,k} A_{b,k} - 0 \right) \right) \\
\text{s.t.} \quad & \sum_{k=1}^{K} X_{b,k} = 1, \quad b = 1, ..., N^t, \quad t = 1, ..., T \quad (2) \\
\quad & \sum_{b=1}^{N^t} X_{b,k} = 1, \quad k = 1, ..., K, \quad t = 1, ..., T \quad (3) \\
\quad & w_k^t \geq (t - 1) \times CT, \quad k = 1, ..., K, \quad t = 1, ..., T \quad (4)
\end{align*}
\]

where

- \( \Pi \) : set of sequence (\( \pi \in \Pi \)),
- \( P_k^t \) : line stoppage time by the part shortage in workstation \( k \) at injection period \( t \) (\( t = 1, ..., T \)),
- \( A_{b,k} \) : line stoppage time by the work delay of RM in workstation \( k \) at injection period \( t \),
- \( D_k^t \) : idle time of worker in workstation \( k \) at injection period \( t \),

\( R' \) : idle time of RM whose work is not assigned at injection period \( t \),

\( G_k^t \) : idle time of RM whoes finishing time is earlier than \( CT \) in workstation \( k \) at injection period \( t \),

\( S' \) : set of workstation that needs RM at injection period \( t \) (\( t \leq K \)),

\( w_k^t \) : work starting time of RM in workstation \( k \) at injection period \( t \),

\( X_{b,k} \) : 0-1 variable that takes 1 if RM \( b \) is assigned to workstation \( k \) at injection period \( t \), otherwise, 0,

\( \rho \) : weighting factor (\( 0 < \rho < 1 \)).

The objective function Eq.(1) is given as the weighted sum of the line stoppage time and the idle time each of which will be described below in detail.

Figure 2 illustrates a feature that the part shortage will happen when the quantity of part \( m \) used actually before the injection period \( t \) (\( \sum_{i=1}^{N} a_{m,i} x_{i}^{t} \)) exceeds its ideal quantity (\( r_{m}^{t} \)). In this case, \( P_k^t \) is given as Eq. (5).

\[
P_k^t = \max_{1 \leq m \leq M} \left( \max_{1 \leq i \leq N} \left( \sum_{b=1}^{N^t} a_{m,i} x_{b,k} - r_{m}^{t} \right) / \prod_{l=1}^{t-1} \right), \quad (5)
\]

where \( a_{m,i} \) is the quantity of part \( m \) required per model \( i \); \( x_{b,k}^{t} \) is the cumulative amount of production for model \( i \) during injection period from 1 to \( t \), and \( r_{m}^{t} \) is the ideal usage rate of part \( m \). In Fig. 3, let \( m^{*} \) (see Eq.(6)) be the travelling time of RM \( b \) who moves from workstation \( s \) to \( k \) during injection period from \( t-1 \) to \( t \), and \( W_{m}^{t-1} \) and \( W_{m}^{t} \) the working time of RM \( b \) on \( s \) at \( t-1 \) and \( k \) at \( t \), respectively. \( A_{b,k}^{t} \) is given by Eq.(7):
\[ m_{i,j,k} = |WE_{i,j} - WS_{i,j}| \times O. \]  
\[ A'_i = \max(m_{i,j,k} + WL_{i,j}, W_{i,j,k} - 2CT, 0), \]  
\[ \forall j \in S_{i-1}, \ b = 1, 2, ..., N'. \]  

where \( WE_{i,j} \) denotes the completion time of work on the workstation \( j \) at the injection period \( i - 1 \), and \( WS_{i,j} \) the start time of work on \( j \) at \( i \), and \( O \) the travel time of \( RM \) per workstation. The idle time of worker occurs when the work time denoted by \( L_{i,j} \) on workstation \( j \) at injection period \( i \) exceeds \( LT \). Consequently, \( D'_i \) is given as Eq.(8).

\[ D'_i = \max(CT - L_{i,j}, 0), \]  

Moreover, the idle time of \( RM \) is classified into two categories, i.e., that of \( RM \) (\( R' \)) and of standby \( RM \) (\( G'_i \)). The former is given by Eq.(9) where \( N' \) denotes the \( RM \) number working at the injection period \( i \) and \( B \) the maximum number of \( RM \) necessary throughout the production planning horizon. The latter is equivalent to the idle time of \( RM \) \( b \) who cannot begin the work yet at the injection period \( i \) even though he or she has reached the workstation \( j \) from \( s \) (see Figure 4), and given by Eq.(10).

\[ R' = (B - N')CT, \]  
\[ G'_i = \max(\max(CT - m_{i,j,k} - WL_{i,j}, 0), \forall s \in S_{i-1}, \ b = 1, 2, ..., B). \]  

On the other hand, sequence \( \pi \) and 0-1 variable \( x_{i,j,k} \) are the decision variables of this problem. Among the constrains, Eqs.(2) and (3) are the conditions of generalized assignment, and Eq.(4) is on the starting time of \( RM \)'s work.

**SOLUTION METHOD**

Since the above combinatorial problem becomes NP hard, an approximated method is more desirable than the rigid one for practical applications. From this aspect, we have developed a hybrid method that divides the original problem into two subproblems in a hierarchical manner so that we can apply the efficient solution methods for each of them. Its upper level solves a sequencing problem based on SA (Simulated Annealing), and the lower a route selection problem of \( RM \) using NNM (Nearest Neighbor Method; Yanaura and Ibaraki, 2001). Moreover, we apply the goal chasing method to derive an initial sequence for effective search in SA, and the levering method to decide the \( RM \) number. Below, each algorithm will be explained in detail (see Figure 5).

\[ \text{Start} \rightarrow \text{Generation of initial sequence} \rightarrow \text{Renovation of sequence} \rightarrow \text{End} \]

\[ \rightarrow \text{Decision of RM number} \rightarrow \text{No} \rightarrow \text{Convergence?} \rightarrow \text{yes} \rightarrow \text{Decision of RM's route} \]

\[ \rightarrow \text{Decision of RM's route} \rightarrow \text{lower level (using the nearest neighbor method)} \rightarrow \text{Decision of RM's route} \]

\[ \rightarrow \text{upper level (using SA)} \rightarrow \text{Decision of RM's route} \rightarrow \text{Decision of RM's route} \]

\[ \rightarrow \text{(using the goal chasing method)} \rightarrow \text{Decision of RM's route} \rightarrow \text{Decision of RM's route} \]

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\[ \rightarrow \text{(using the goal chasing method)} \rightarrow \text{Decision of RM's route} \rightarrow \text{Decision of RM's route} \]

**[Proposed Algorithm]**

Step 1 (upper level): Generate the initial sequence based on the goal chasing method.

Step 2 (lower level): Calculate the maximum number of \( RM \) required throughout production planning period after

**Figure 3.** Line stoppage due to work delay of \( RM \)

**Figure 4.** Idle time of \( RM \)

**Figure 5.** Proposed solution method

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smoothing the workloads under the given sequence by the levering method (LM).

Step3(upper level): Decide the travelling route of RM using nearest neighbor method (NNM).

Step4(upper level): Check the convergence condition, and stop if satisfied. Otherwise, generate a new sequence (solution) using the swap neighborhood, and go back to Step2.

To decide the route of RM in the above Step3, we use the following algorithm based on NNM.

[Route Selection Algorithm]

Step3-1: Decide workstations $k_j^*(\in S_j^*, j = 1, 2, \ldots)$ where the assist of RM is necessary per each injection period under the given sequence. Allocate each RM at random to $k_j^*$ (t=1).

Step3-2: Let t=t+1. After making each pair of RM and $k_j^*$, $p_j^*(b = 1, 2, \ldots, B)$, generate every combination $C_i^v (v = 1, 2, \ldots) (p_j^*p_j^* (x, y = 1, 2, \ldots); b \neq x, j \neq y)$. 

Step3-3: Evaluate each $C_i^v$ on the basis of objective function value, and decide the assignment based on the combination having the minimum value. Here, when there exist multiple minima, decide one according to the following priority.

[Priority from the top to the bottom]

(1) One with the smallest sum of RM’s travelling time from $t - 1$ to $t$.
(2) One whose work start time of RM at $t$ is nearest to $t \cdot CT$. 
(3) Arbitrary.

Step3-4: If $t = T$, stop. Otherwise, go back to Step3-2.

NUMERICAL EXPERIMENTS

Numerical experiments are taken place under conditions shown in Tables 1 and 2. We also give the following conditions about the SA parameters: parameter of initial temperature is 0.95; annealing ratio 0.95; number of search per every temperature 100; the number of annealing 100. Moreover, the weighting factor $\rho$ in Eq.(1) is set at 0.9. And we evaluated the results on the basis of average over 100 data sets generated randomly.

Effect of using RM

To examine the effectiveness of using RM regarding the line stoppage time and the value of objective function, we compared the case with that without RM (in Table 3). From Table 3, the assist of RM is known to realize the efficient assembly line management, i.e. both the line stoppage time and the objective function value of the relief case are less than those of the no relief case. This means that leveling the unbalanced workloads is realized every injection period by using RM.

Table 1. Input parameter

| Cycle time[min] | 3 |
| Station number | 20 |
| Product model(A~E) | 5 |
| Total production number | 100 |
| Part number | 10 |
| Part number used per workstation | 0~2 |
| Assembly time per workstation[min] | 1~6 |
| Travelling time of RM per workstation[min] | 0.1 |

Table 2. Production number and assembly times

| Product model | A | B | C | D | E |
| Production number | 10 | 20 | 40 | 20 | 10 |
| Total assembly time[min] | 20 | 52 | 60 | 72 | 120 |

Table 3. Effect of the existence of RM

<table>
<thead>
<tr>
<th>Objective function value, Eq.(1)</th>
<th>With RM</th>
<th>Without RM</th>
</tr>
</thead>
<tbody>
<tr>
<td>132.0</td>
<td>298.2</td>
<td></td>
</tr>
<tr>
<td>$LST^*$ by delayed work[min]</td>
<td>6.4</td>
<td>300.0</td>
</tr>
</tbody>
</table>

* : line stoppage time.

Table 4 shows the change of necessary number of RM according to the increases in total production number of model $E$ that is a product model with the longest assembly time among all models. As supposed beforehand, the necessary RM also increases with the growth of rate of $E$. However, its increasing rate is small compared with that of $E$. From these facts, we know that assigning the appropriate numbers of RM is effective and essential as well for dealing with the production fluctuations.

Table 4. Necessary RM number with the increase in rate of model $E$

<table>
<thead>
<tr>
<th>Rate*</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM number</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

* : Rate of model E in total production number.
Discussion on the proposed solution procedure

Figure 6 shows the convergence process when the goal chasing method is used to generate the initial solution. It has a high speed convergence compared with the random generation. This is because the foregoing search begins with a sequence with rather small rate of the line stoppage.

On the other hand, to verify the effectiveness of the NNM algorithm in deciding the travelling route of RM, we compared the result with the case where the route is generated at random (hereinafter, we call it random generation). Table 5 shows that NNM outperforms the random generation about every aspect, i.e., the objective function value, the line stoppage time, the idle time, and the necessary RM number. This comes from the fact that shortening the travel time can produce the available work time of RM. Moreover, since NNM can reach at a near optimum solution in a minute (1.21[min]), the proposed solution procedure is promising to cope with real-life problems.

CONCLUSION

In this paper, we have considered a sequencing problem of the mixed-model assembly line where the assembly time differs greatly depending on the product-models. After introducing RM as a key factor for the problem-solving, we formulated this problem as a combinatorial optimization problem minimizing the line stoppage time and the idle time simultaneously through the leveling of part usage and the smoothing of workload. Then we have proposed a practical solution method that uses the meta-heuristic methods in a hierarchical manner and can solve the real-life problems in a reasonable computation time. Through numerical experiments, our approach is known to have a great effect on reduction of the line stoppage time and the idle time. Finally, we emphasize another advantage of the proposed approach such that the production changes due to the demand fluctuations for example can be managed just by changing dynamically RM number.

REFERENCES